Expansion-induced contribution to the precession of binary orbits

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Abstract

We point out the existence of new effects of global spacetime expansion on local binary systems. In addition to a possible change of orbital size, there is a contribution to the precession of elliptic orbits, to be added to the well-known general relativistic effect in static spacetimes, and the eccentricity can change. Our model calculations are done using geodesics in a McVittie metric, representing a localized system in an asymptotically Robertson-Walker spacetime; we give a few numerical estimates for that case, and indicate ways in which the model should be improved.

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The issue of whether the global cosmological expansion affects local gravitating systems, such as planetary systems and galaxies, has been studied for a long time (for recent work, and discussions of previous results, see e.g. Refs. [1, 2, 3, 4]). The specific questions asked in the literature focus mostly on the extent to which local systems expand, and on the form and magnitude of the corrections to the effective forces felt by orbiting test bodies; while the approaches used vary, sometimes in conceptually important ways, the consensus is that there is an effect in principle, but in practice it is exceedingly, undetectably small. To a first approximation, a reasonable, physically motivated point of view, as expressed by Misner, Thorne, and Wheeler [5], for the case of galaxies, is that they are like rigid pennies attached to the expanding balloon representing the universe, and they do not themselves expand.

While we agree with this general conclusion, at least in terms of currently feasible observations, an increase in orbit size is not the only effect the cosmological expansion can have on local systems, nor the only cumulative one. In our approach to the search for such effects, we will use as a model spacetime an actual solution of Einstein's equation which represents a gravitating body in an asymptotically expanding universe; restrict our attention to nearly Newtonian orbits in the weak-gravity, slow-expansion approximation; and consider the time variation of the parameters characterizing the corresponding Keplerian ellipses, their size, perihelion angle, and eccentricity.

Before we proceed, however, we will make a few comments on the systems involved. It is well known that Keplerian elliptical orbits in Newtonian gravity do not precess (the $1/r$ potential is one of two that yield closed orbits, together with the harmonic oscillator, as Bertrand's theorem states) but the corresponding ones in general relativity do, while the size and eccentricity of the general relativistic orbits are constant. These results hold for the case of a single, spherically symmetric, isolated center of attraction, with a test body and no other matter around it. Corrections due to other orbiting bodies can be calculated, and they may affect all parameters of the orbits, but as far as the global expansion is concerned, under the above conditions there can be *no local effects* due to cosmological expansion in general relativity, regardless of the rate at which the rest of the universe expands, because Birkhoff's theorem then concludes that locally the spacetime must be the static, Schwarzschild solution (Einstein and Straus [6] also showed this explicitly in one class of models, without using Birkhoff's theorem). The cosmological expansion can be felt by a local system only if (i) The situation is not spherically symmetric, and/or (ii) There is matter (some gas, dust, or dark matter, say) within the local system, or a cosmological constant. This is one of the many ways in which one can see that general relativity obeys the spirit of Mach's principle, but only up to a point (see, e.g., Ref. [7]); the global behavior of distant objects affects the local dynamics, but the dynamical equations are local.

In a realistic model of a local system, both deviations from spherical symmetry and extra matter will have an effect; in the solar system, for example, the presence of other stars, solar multipoles such as oblateness, minor objects, interplanetary matter and/or the solar wind and radiation, are ways in which this happens. Studying the way the global expansion affects the local system then means: (i) Understanding how the dynamics of surrounding non-symmetric matter and/or the local internal environment of the system depends on the global behavior of the universe, and (ii) Calculating the effect of the local environment on the dynamics of the system of interest. Notice that, for a local system that is considered as part of a larger one, such as a planetary system inside a galaxy, the first part amounts to assuming that a similar problem has been solved one level up; for example, the spacetime metric for a planetary system should not be asymptotic to a Robertson-Walker metric expanding at the Hubble rate, but to a metric that is appropriate inside the galaxy, expanding, if at all, at a rate previously found for that system. The problem thus becomes somewhat involved, but it seems to us that this is the only way to avoid sneaking in an assumption about the local environment's expansion as part of the setup, which has occasionally been done. On the other hand, once it has been formulated in this way, the problem is broken down into parts that can be separately investigated, and labelled by the structure scale under consideration (galactic, planetary, ...), mechanism by which the effect may be transmitted, and type of effect.

In this letter, as a start, we will limit ourselves to considering a simplified model to show the effects that may arise. Spherical symmetry will be preserved, and the mechanism responsible for transmitting environment effects will be the presence of matter in the local system. In our numerical estimates we will use orbital parameters relevant for a couple of known planets and galaxies, and the Hubble constant to obtain the environment expansion rate; much more work will be needed on several aspects of this problem, including the type of matter and spacetime model used and the role of anisotropy, before we can claim to have a reasonable understanding of it along the lines described above.

While spherically symmetric, vacuum, asymptotically flat spacetimes and homogeneous, isotropic cosmological ones with fluid matter or a cosmological constant can be easily treated in general relativity and give rise, respectively, to the Schwarzschild solution and the Friedmann-Robertson-Walker or de Sitter spacetimes, solutions representing an isolated massive object embedded in an expanding universe are more difficult to get, and fully explicit forms are not usually given (for a good general discussion, see Ref. [8]).

One family of solutions representing isolated bodies in asymptotically FRW spacetimes, for the cases $k = 0, \pm 1$, was given in the 1930's by McVittie [9]. The (asymptotically) spatially flat model $k = 0$, in the isotropic coordinate form used by Hogan [10], has line element

$$\mathrm{d}s^2 = -\left(\frac{1 - M\mathrm{e}^{-\beta/2}/2r}{1 + M\mathrm{e}^{-\beta/2}/2r}\right)^2 \mathrm{d}t^2 + \mathrm{e}^{\beta(t)}\left(1 + \frac{M\mathrm{e}^{-\beta/2}}{2r}\right)^4 (\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2)\,. \quad (1)$$

Here, the constant M is interpreted as the "mass at the singularity," and $\beta(t)$ represents the (asymptotic) expansion rate of the universe. We will not give an explicit expression for $\beta(t)$, since it is related to the specific equation of state of the fluid, but will think of it instead in terms of an expansion $\beta(t) = \beta_0 + \dot\beta_0\,(t - t_0) + \mathcal{O}((t - t_0)^2)$, around the present time t_0, say, where the first few coefficients can be fitted phenomenologically.

The above line element has the advantage that, as $r \to \infty$, the $t =$ const hypersurfaces become the surfaces of homogeneity of a Robertson-Walker model. For our purposes however, we find it more convenient to work with a radial coordinate with a geometrical meaning tied to the area of the corresponding 2-sphere, and use $R := r\,\mathrm{e}^{\beta/2}(1 + GM\mathrm{e}^{-\beta/2}/2r)^2$ in the interval $R \in (2M, \infty)$ [4]. We can then rewrite the line element as

$$\mathrm{d}s^2 = \left[-f(R) + \left(\frac{R\dot\beta}{2c}\right)^2\right]c^2\mathrm{d}t^2 + \frac{2}{\sqrt{f(R)}}\frac{R\dot\beta}{2c}\,c\mathrm{d}t\,\mathrm{d}R + \frac{\mathrm{d}R^2}{f(R)} + R^2\,\mathrm{d}\Omega^2\,, \quad (2)$$

where $f(R) := 1 - 2GM/(c^2R)$, and we have restored all c's and G's.

The use of the McVittie metric as a model for studying local systems in RW universes has been the subject of criticism [11, 12], but in a series of papers [13, 4, 14] Nolan made a good argument for the $k = 0$ model by studying its global properties in detail. Nolan also showed that, in an appropriate sense, those solutions are the unique ones representing an isolated mass surrounded by a shear-free perfect fluid in a spatially flat FRW universe; the shear-free condition leads to special propertie that we would not expect a black hole to have [4], but the metric does provide us with a viable explicit model for our purposes.

Because of the spherical symmetry, we consider as usual our orbits to lie in the $\sin\theta = 1$ plane, and drop θ from the whole treatment from now on. For a particle of mass m moving in the metric (2), one way of deriving the equations of motion is to use the Hamiltonian approach. The canonical momenta obtained from the action $S[x] = (m/2)\int \mathrm{d}\tau\, g_{\mu\nu}\,\dot x^\mu \dot x^\nu$ are $p_\mu = m\,g_{\mu\nu}\dot x^\nu$, where the overdot on a dynamical variable, $\dot t$, $\dot R$, or $\dot\phi$, denotes a derivative with respect to proper time τ along a trajectory (whereas $\dot\beta$ just indicates the derivative of the known function $\beta(t)$ with respect to its argument).

The test particle Hamiltonian $\mathcal{H} = (2m)^{-1} g^{\mu\nu} p_\mu p_\nu$ in this case becomes

$$\mathcal{H} = \frac{1}{2m}\left(-\frac{p_t^2}{f(R)\,c^2} + \frac{2}{\sqrt{f(R)}}\frac{R\dot\beta}{2c}\,\frac{p_t p_R}{c} + \left[f(R) - \left(\frac{R\dot\beta}{2c}\right)^2\right] p_R^2 + \frac{p_\phi^2}{R^2}\right),$$

supplemented by the constraint that $g^{\mu\nu} p_\mu p_\nu = -(mc)^2$, or $\mathcal{H} = -\frac{1}{2}\,mc^2$.

Angular momentum is conserved, $p_\phi = L$, and as is common in celestial mechanics orbit problems, we reexpress R in terms of $u := 1/R$ in the equations of motion. If we perturbatively expand those equations in powers of $1/c$ and $\dot\beta$, zeroth order terms correspond to a particle in Newtonian gravitation; neglecting all $\dot\beta$ terms corresponds to doing calculations in a static spacetime, and we recover the well-known Schwarzschild results; the expansion effects we are interested in arise when leading order terms in $\dot\beta$ are kept.

The radial equation can be recast into an orbit equation in terms of $u(\phi)$,

$$u'' + u = \frac{GMm^2}{L^2} + \frac{3\,GMu^2}{c^2} - \frac{\dot\beta^2 m^2}{4L^2u^3} - \frac{GM\dot\beta m}{2c^2Lu}\,u'\,.$$

If we look for a solution of the form $u(\phi) = u_0 + \xi(\phi)$, where $u_0 = GMm^2/L^2$ corresponds to a Newtonian circular orbit and $\xi(\phi)$ is a small perturbation (see, e.g., Refs. [15, 16]), the linearized equation for the deviation $\xi(\phi)$ is

$$\xi'' = \frac{3\,GMu_0^2}{c^2} - \frac{\dot\beta^2m^2}{4L^2u_0^3} - \left(1 - \frac{6GMu_0}{c^2} - \frac{3\dot\beta^2m^2}{4L^2u_0^4}\right)\xi - \frac{GM\dot\beta m}{2c^2Lu_0}\xi'\,. \tag{3}$$

It should be noted that for a general McVittie metric, $\dot\beta = \dot\beta(t(\phi))$, so, to simplify the solution of the equation, we will now limit ourselves to considering the case $\ddot\beta = 0$; in terms of comoving coordinates as in Eq. (1), this is the linear expansion case, although the metric for this model has a timelike Killing vector field, as can be seen from the fact that the line element (2) becomes time-independent. Equation (3) is then of the form $\xi'' = f_\mathrm{c} - \omega^2\,\xi + f_\mathrm{d}\,\xi'$, which can be viewed as that of an oscillator with a constant force f_c, modified by a $\dot\beta$ term which affects the equilibrium displacement of the oscillator and thus the geodesic orbit size, but it does *not* lead to orbit expansion when $\dot\beta$ is time-independent; the frequency ω is also modified by a $\dot\beta$ term, which contributes to the orbital precession, and, interestingly, there is a damping term f_d, which leads to a change in orbital eccentricity. The equation admits a general solution of the form $\xi(\phi) = \mathcal{C} + \mathcal{E}\mathrm{e}^{\mathcal{B}\phi}\cos(\mathcal{A}\,\phi - \phi_0)$, depending on arbitrary constants $\mathcal{E}$ and ϕ_0, and where

$$\mathcal{A} := \sqrt{\omega^2 - \tfrac{1}{4}\,f_\mathrm{d}^2} \approx 1 - \frac{3GMu_0}{c^2} - \frac{3\dot\beta_0^2m^2}{8L^2u_0^4}\,,$$

$$\mathcal{B} := \tfrac{1}{2}\,f_\mathrm{d} = -\frac{GM\dot\beta_0 m}{4c^2Lu_0}\,,$$

$$\mathcal{C} := \frac{f_\mathrm{c}}{\omega^2} \approx \frac{3GMu_0^2}{c^2} - \frac{\dot\beta_0^2m^2}{4L^2u_0^3}\,,$$

from which, defining $u_0' = u_0 + \mathcal{C}$ and $\varepsilon_0 = \mathcal{E}/(u_0 + \mathcal{C})$, we get

$$u(\phi) = u_0' \big[1 + \varepsilon_0 \, \mathrm{e}^{\mathcal{B}\phi} \cos(\mathcal{A}\,\phi - \phi_0)\big] \,. \tag{4}$$

The angle σ by which the orbit (4) precesses during each revolution can be found from the fact that the change $\Delta\phi$ such that the argument of the cosine increases by 2π is $2\pi + \sigma = 2\pi/\mathcal{A}$; i.e., for $\mathcal{A} \approx 1$, $\sigma \approx 2\pi\,(1 - \mathcal{A})$, or

$$\sigma \approx \frac{6\pi GM}{a(1-\varepsilon^2)\,c^2} + \frac{3\pi}{4}\,\frac{\dot\beta_0^2 a^3 (1-\varepsilon^2)^3}{GM}\,, \tag{5}$$

where $\varepsilon = \varepsilon_0 \, \mathrm{e}^{\mathcal{B}\phi}$ is the orbital eccentricity, and we have used the fact that for a Keplerian ellipse $1/u_0 = a\,(1-\varepsilon^2)$, with a the semi-major axis. The first term in (5) is the Schwarzschild contribution, σ_0, as expected. The eccentricity also changes, by a fractional amount per revolution $\Delta\varepsilon/\varepsilon = \mathrm{e}^{2\pi\mathcal{B}} - 1$, or

$$\frac{\Delta\varepsilon}{\varepsilon} \approx -\frac{\pi GM\dot\beta_0 m}{2c^2 L u_0} = -\frac{\pi\dot\beta_0}{2c^2}\,\sqrt{GMa\,(1-\varepsilon^2)}\,. \tag{6}$$

Equations (5) and (6) are our main results; they show that, even in a model with no orbit expansion (this may just be a feature of the $\ddot\beta = 0$ McVittie models used), global expansion has an effect on periastron precession and eccentricity.

We can get a first crude estimate of the magnitude of these effects by evaluating σ and $\Delta\varepsilon/\varepsilon$ for a few known systems. For the solar planets Mercury and Pluto, even using a Hubble parameter value H_0 around 75 km/s/Mpc for $\dot\beta_0$, which is certainly a vast overestimate, one obtains for the relative size of the expansion-induced precession term in (5), values $\sigma_\beta/\sigma_0 \approx 3 \times 10^{-21}$ and 7×10^{-13}, respectively, and for the relative eccentricity change in (6), -1.5×10^{-21}/rev and -1.5×10^{-20}/rev, respectively. Stronger gravitational fields from higher values of M and/or smaller values of a only make things worse, and we conclude that all expansion effects are negligible for systems in which general relativistic effects have been studied already and have small orbital periods, allowing us to observe many revolutions (such as the binary pulsar [17]); in other words, within the approximations of our model, none of these effects is relevant for the astrophysics of stellar systems.

What about stars revolving around galactic centers, or gravitationally bound pairs of galaxies? After all, as one might expect from cosmological effects (and in contrast to the Schwarzschild ones), σ_β/σ_0 and $\Delta\varepsilon/\varepsilon$ grow with orbit size a. We can get an idea if we apply equations (5) and (6) to the Large Magellanic Cloud, at $a \approx 20$ kpc from the Milky Way, with $M \approx 10^{11}\,M_\odot$, which gives $\sigma_\beta/\sigma_0 \approx 9 \times 10^{-3}$ and $\Delta\varepsilon/\varepsilon \approx -5 \times 10^{-11}$/rev, several orders of magnitude larger than the previous values, in a regime where the use of H_0 for $\dot\beta$, although probably still not appropriate, comes closer to being realistic.

So, are our expansion effects cosmologically relevant? The fact that the Hubble distance-redshift relation shows local deviations, and our speed away from the Virgo Cluster is affected by a gravitational pull, shows that there are scales at which local interactions and global expansion coexist. Similar ideas have already motivated, e.g., work on the effect of global expansion on the formation and evolution of clusters of galaxies [18], and more recently on the effect of inhomogeneities on the overall expansion [19] or of a cosmological constant on local dynamics [20]. The present approach can be seen as a model for the onset of this situation with simple, binary systems. The model should now be improved to include more realistic source masses and anisotropy, as well as possible multipole moments of the orbiting mass [21]. Even then the numbers may be small, especially if one thinks about the time scales involved in galactic dynamics, but they may lead to interesting statistical effects if a large number of galaxy pairs are observed.

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